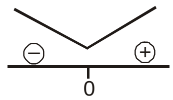
EXERCISE – II HINTS & SOLUTIONS

Sol.1 D

$f(x) = \dfrac{x^2 - 1}{x^2 + 1}$



$f'(x) = \dfrac{4x}{(x^2 + 1)^2}$ x = 0 is minima

$f(0) = -1$

Sol.2 A,C,D

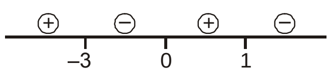
$f(x) = 40\,(3x^4 + 8x^3 - 18x^2 + 60)^{-1}$

$f'(x) = -40\,(3x^4 + 8x^3 - 18x^2 + 60)^{-2}\,(12x^3 + 24x^2 - 36x)$

$f'(x) = 0$

$\Rightarrow\ 12x(x^2 + 2x - 3) = 0$

$12x\,(x + 3)\,(x - 1) = 0$



x = –3, 1 Local Maxima

x = 0 Local minima

Sol.3 B

$f(x) = a\,\ell n\,|\,x\,| + bx^2 + x$

$f'(x) = \dfrac{a}{x} + 2bx + 1$

$f'(-1) = 0 \Rightarrow -a - 2b + 1 = 0$ .....(1)

$f'(2) = 0 \Rightarrow \dfrac{a}{2} + 4b + 1 = 0$ .....(2)

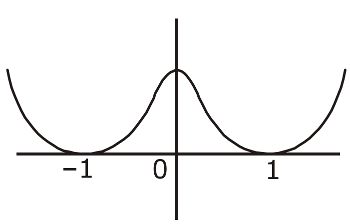
Solving (1) & (2)

we get a = 2, b = –1/2

Sol.4 A,C,D

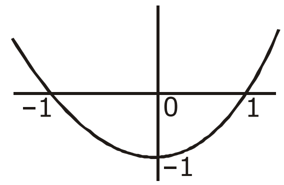
If n = 2

$f(x) = (x^2 - 1)^2\,(x^2 + x + 1)$



If n = 3

$f(x) = (x^2 - 1)^3\,(x^2 + x + 1)$



Sol.5 A,C

$f'(x) = \dfrac{3(\sin^{-1} x)^2}{\sqrt{1 - x^2}} - \dfrac{3(\cos^{-1} x)^2}{\sqrt{1 - x^2}} = 0$

$\sin^{-1} x = \cos^{-1} x \Rightarrow x = \dfrac{\pi}{4}$

critical points are $x = 1, -1, \dfrac{\pi}{4}$

$f\left(\dfrac{\pi}{4}\right) = \dfrac{\pi^3}{32}$;

$f(-1) = -\dfrac{\pi^3}{8} + \pi^3 = \dfrac{7\pi^3}{8}$

Sol.6 B,D

$y = f(x) = \dfrac{x}{1 + x\tan x}$

y_{max} when is reciprocal take min. value

$\dfrac{1}{y} = \dfrac{1 + x\tan x}{x} = \dfrac{1}{x} + \tan x$

Let $z = \dfrac{1}{x} + \tan x$

$\dfrac{dz}{dx} = -\dfrac{1}{x^2} + \sec^2 x = 0 \Rightarrow x = \cos x$

$\dfrac{d^2z}{dx^2} = \dfrac{2}{x^3} + 2\sec^2 x \tan x > 0$

min. at x = cos x

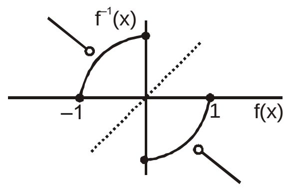
y take max. value at some x_0

where $x_0 = \cos x$

Sol.7 A,C

$f(X) = -\sqrt{1-x^2}\ ,\qquad 0 \le x \le 1$

$\qquad -x \qquad\qquad x > 1$



Sol.8 B,D

$x = \phi(t) = t^5 - 5t^3 - 20t + 7$

$y = \psi(t) = 4t^3 + 4t^2 - 18t + 3$

$\dot{x} = \phi'(t) = 5t^4 - 15t^2 - 20$

$= 5\,(t^2 - 4)\ (t^2 + 1)$

$\dot{x} \ne 0$ when $-2 < t < 2$

$\dot{y} = \psi'(t) = 12t^2 + 8t - 18$

$= 6\,(2t^2 - t - 3)$

$= 6\,(2t - 3)\,(t + 1)$

$\dot{y} = 0 \Rightarrow t = 3/2,\quad t = -1$

$-2 < t < 2$ satisfied

$$\frac{dy}{dx} = \frac{\dot{y}}{\dot{x}} = 0 \qquad (x \ne 0)$$

$\dot{y} = 0 \Rightarrow t = -1,\ 3/2$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{\dot{y}}{\dot{x}}\right) = \frac{d}{dt}\left(\frac{\dot{y}}{\dot{x}}\right).\frac{dt}{dx}$$

$$= \frac{\dot{x}\,\ddot{y} - \dot{y}\,\ddot{x}}{(\dot{x})^2}\cdot\frac{1}{(\dot{x})}$$

$$\frac{d^2y}{dx^2} = \frac{\dot{x}\,\ddot{y}}{\dot{x}^2} \times \frac{1}{\dot{x}}$$

$$\frac{d^2y}{dx^2} = \frac{\ddot{y}}{\dot{x}^2} = \frac{6(4t-1)}{\dot{x}^2}$$

at $t = -1$ $\qquad \dfrac{d^2y}{dx^2} < 0$

at $t = 3/2$ $\qquad \dfrac{d^2y}{dx^2} > 0$

$t = -1 \quad y_{max} = 14$

$t = 3/2 \quad y_{min} = -\dfrac{69}{4}$

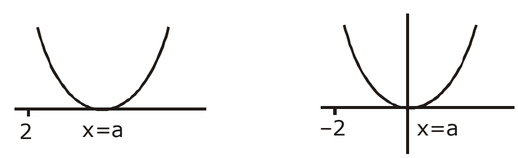
Sol.9 A,C

$f(x) = x^{2/3}$

$$f'(x) = \frac{2}{3.x^{1/3}}$$

$f'(x)$ changes its sign near about origin and at origin does not exist

Sol.10 A,D

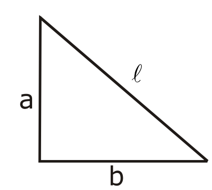


Sol.11 A,C

In all three definition separately differentiate & check.

Sol.12 A,C

$a + b = 9$



$v = \dfrac{1}{3}\pi r^2 h$

$= \dfrac{1}{3}\pi b^2 a = \dfrac{1}{3}\pi b^2 (9 - b)$

$\dfrac{dv}{db} = 0 \Rightarrow b = 6 \Rightarrow a = 3$

$\ell = \sqrt{a^2 + b^2} = 3\sqrt{5}$

Surface area $= \pi r \ell$

$= \pi b \ell$

$= \pi (6)(3\sqrt{5}) = 18\sqrt{5}\,\pi$

Sol.13 A,B,C

f(x) = sin x – x cos x

f ′(x) = cos x + x sin x – cos x

f ′(x) = x sin x = 0

$\Rightarrow$ x = 0 sin x = 0

(reject) x = nπ

f ′(x) = sin x + x cos x

x = π f ″(π) = – π < 0 max.

x = 2π f ″(2π) = 2π > 0 min.

x = –π f ″(–π) = π > 0 min.

x = –2π f ″(–2π) = – 2π < 0 max.

Sol.14 A,B,D

$$f(x) = \frac{x+1}{x^2+1}$$

$(x^2 + 1)$ f ′(x) + 2x f(x) = 1

$$f'(x) = \frac{3x^2+2x+1}{(x^2+1)^2} \neq 0$$

$$f''(x) = \frac{x^3+3x^2-3x-1}{(x^2+1)^4} = 0$$

$x^3 + 3x^2 – 3x – 1 = 0$

$(x – 1)(x^2 + 4x + 1) = 0$

x = 1, $(x^2 + 4x+1)=0$ $\Rightarrow x = -2 \pm \sqrt{3}$

Sol.15 B,C

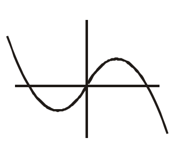
odd cubic polynomial

f(x) = ax^3 + cx

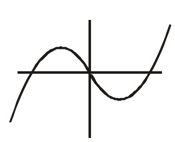
f′(x) = $3ax^2$ + c

Two points possible

only when a, c have opposite sign.



or



Sol.16 A,C,D

check for

f(1 + x) = f(1 – x)

& f(2 + x) = f(2 – x)

f″(1) < 0

$\Rightarrow$ x = 1 is point of maxima.

Sol.17 B,C

$$y = \frac{ax^2+2bx+c}{Ax^2+2Bx+C}$$

Cross multiply

$ax^2 + 2bx + c - y(Ax^2 + 2Bx + C) = 0$

D $\geq$ 0 $\forall$ x $\in$ R

Let y $\in$ [α, β]

Equality holds when D = 0

$\Rightarrow$ α, β are the extremum values

Sol.18 A,C

From options we can check by putting the value in function instead of differentiatig the function.

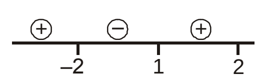
Sol.19 A,C,D

$$f(x) = \log(x-2) - \frac{1}{x}$$

$$f'(x) = \frac{1}{x-1} + \frac{1}{x^2} = \frac{x^2+x-2}{x^2(x-2)} = 0$$

$x^2 + x – 2 = 0$

x = 1, –2



$\uparrow$ (2, ∞)

$$f''(x) = -\frac{1}{(x-2)^2} - \frac{2}{x^3} < 0$$

concave downwards always

f^{-1} (x) will also $\uparrow$ whenever defined